disc 106

Wednesday, February 17, 2021 1:03 PM

Math 128a – Week 5 Worksheet
GSI: Izak, (2/17/21)

2.3 Problems

Problem 1. *Derive the error formula for Newton's method:*

$$|p - p_{n+1}| \leq \frac{M}{2|f'(p_n)|}|p - p_n|^2$$

2.5 Problems

Problem 2. *Steffensen's method is applied to a function $g(x)$ using $p_0^{(0)} = 1, p_2^{(0)} = 3$ to obtain $p_0^{(1)} = .75$. What is $p_1^{(0)}$?*

2.6 Problems

Problem 3. *Use Horner's method to evaluate $P(x) = 7x^4 - 2x^2 - 5x - 3$ at $x = 1$*

3.1 Problems

Problem 4. *Given $f(x) = x^3 - 4x^2 + 4$, find the Lagrange interpolation polynomial of degree at most three using the nodes $x_0 = -3, x_1 = -1, x_2 = 1, x_3 = 5$*

Problem 5. *Let $x_0 = -1, x_1 = 0, x_2 = 1$, define $f_0(x) = x^2 - 1, f_1(x) = 2x^2 + 3x, f_2(x) = -x^2 + 2x$. Evaluate these polynomials at x_i. Uses this to find a polynomial of degree at most 2 such that $g(x_0) = -4, g(x_1) = -1$, and $g(x_2) = 6$ without preforming any tedious computations.'*

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4) $L_0(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}$

$L_3(x) =$

$f(x_0) =$
$f(x_1) =$
$f(x_2) =$
$f(x_3) =$

$L_0(x_0) = 1$
$L_0(x_1) = 0$
$L_0(x_2) = 0$
$L_0(x_3) = 0$

$L_i(x_j) = \delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

$f(x_0)L_0(x) + f(x_1)L_1(x) + \dots = x^3 - 4x^2 + 4$

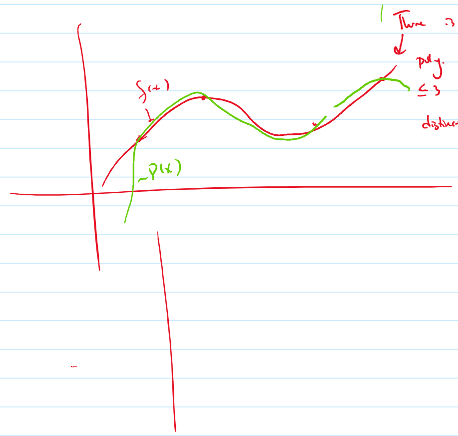
2) $p_0^{(1)} = \Delta^2(p_0^{(0)}, \underset{:= x}{p_1^{(0)}}, p_2^{(0)})$

$.75 = 1 - \frac{(x-1)^2}{3 - 2x + 1}$

Solve for $x \Rightarrow x = 1.5$ & $x = 0$

3)
$$\begin{array}{c|ccccc} 1 & 7 & 0 & -2 & -5 & -3 \\ & \downarrow & 7 & 7 & 5 & 0 \\ \hline & 7 & 7 & 5 & 0 & \boxed{-3 = P(1)} \end{array}$$

$P(x) = (x-1)(7x^3 + 7x^2 + 5x) - 3$



There is a unique poly. of degree ≤ 3 through 4 distinct points.

	x_0	x_1	x_2
f_0	0	-1	0
f_1	-1	0	5
f_2	-3	0	1
g	2	-1	6

$g(x) = f_0(x) + f_1(x) + f_2(x)$

$g = af_0 + bf_1 + cf_2$

$2 = -b - 3c$

$-1 = -a$

$6 = 5b + c$

$$\underbrace{\begin{pmatrix} 0 & -1 & -3 \\ -1 & 0 & 0 \\ 0 & 5 & 1 \end{pmatrix}}_{A} \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 6 \end{pmatrix}$$

	x_0	x_1	x_2
L_0	1	0	0
L_1	0	1	0
L_2	0	0	1